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Abstract

Public debt has been a widespread problem for mainstream economists have no effective procedure to deal with it and prevent its financial, economic and social negative consequences. This paper argues that the public debt is a macroeconomic phenomenon and not the simple company's accounting situation mainstream economists imagine and suggests a new approach to the public debt trend based on the difference equation method. The main hypothesis is that public debt has a negative effect on the GDP and tax receipts while its interest expenses expand private credit supply and thus GDP and tax receipts thus providing extra revenue enough to pay the interest on the public debt. However, public debt, interest expenses and credit supply attained so high levels while consumer's income expanded slowly that the positive effect through private credit is disappearing and no longer can compensate the negative effect of the public debt itself on the tax receipts. Consequently, the public debt may be causing more deficits and more public debt. It may have been thus created a positive feedback process that could lead the public debt to follow an explosive trend. The time trend of the public debt is given by the interest rate on Treasury bonds and the coefficient that measures the negative effect of public debt on tax revenue; the public debt time trend does not depend on the primary surplus. Therefore, austerity programs do not lead the public debt to the desired stability. An experiment applied to the United States in the period 1960-2007 does not allow for the rejection of this hypothesis. The conclusion is that insisting on austerity will lead to more financial crises; a new theoretical approach is required.

Keywords

Public debt, interest expenditure, credit, tax revenue, public debt trend.

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Public Debt Is Economic Nonsense.

Gerson P. Lima¹

1. Introduction

Public debt is a macroeconomic fact and not a simple company's accounting problem as the mainstream economists see it. Accordingly, this paper describes a theorem based on the difference equation method to demonstrate that the public debt may have a theoretical equilibrium solution. However, this solution may be unattainable since monetary policy may preclude the convergence to equilibrium in the real world. In order to test the adherence of this theorem to the reality it is presented an experiment, the estimate of the US federal tax revenue in a non-mainstream aggregate supply and demand approach. Statistical results obtained do not suggest rejecting the hypothesis that it may be expected a negative combined effect of the public debt and interest expenditure on the tax revenue. The main conclusion is therefore that monetary policy creates a positive feedback process that leads the public debt to follow an explosive trend to infinity. Economic, financial and social consequences of such a public debt trend are not different from what has been reported on economics and finance the world around.

Public debt has been a widespread problem for mainstream economists have no effective procedure to deal with it and prevent its economic and social negative consequences. They are convinced that stabilizing the public debt ratio to GDP is the right thing to do and create sophisticated accounting methods to calculate the primary surplus that supposedly provides the targeted public debt ratio stability. Testing hypotheses against reality and the feasibility and consequences of such a primary surplus were until recently not considered in the mainstream monetarist approach. Some progress has been done since the failure of mainstream models to deal with the crises they create, and especially after Krugman's post at The New York Times (Krugman, 2010). Many mainstream economists are considering the idea that actually the public debt may follow an explosive trend. However, mainstream economists cannot provide a solution for the problem since instead of searching into the public debt performance they keep doing the same accounting exercise, looking for a primary surplus that under convenient assumptions would stabilize the ratio public debt/GDP.

The paper is organised as follows. Section 2 brings a critical appreciation of the mainstream methodology applied to the public debt. Section 3 shows the estimate of the US GDP, stressing the effects of the monetary policy on the tax base. Section 4 presents the estimate of the US federal tax revenue both as a reduced equation in a simultaneous equations model and as a function of GDP. The return coefficients of the fiscal and the monetary policies on the US tax receipts are remarked. Section 5 develops the theorem of the public debt time trend and discusses the probability of attaining the necessary condition for debt equilibrium. Finally, Section 6 offers some conclusions.

2. The failure of the mainstream approach to the public debt time trend

Governments are neither profit seeking companies nor common consumers; they are social institutions that sometimes behave as irresponsible consumers. The right lecture of the economics' paradigm by governments seems to be "governments shall never spend more than

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their earnings”. This does not mean that governments should save and harm society; this means that governments can raise their earnings to increase social expenditures and people’s income. Governments can do that at will as their sources of earnings are peculiar: they are legally empowered to collect taxes and to print money. Notwithstanding, congresses of democratic countries transferred the power of money printing to private central bankers who imposed their paradigm: governments cannot print money. Who has the power of printing money rules the economy; so, mainstream economics created the monetary policy to justify the public debt. After decades this arrangement led to many financial crises, but beyond not eliminating crises all mainstream proposals have been turning around the same: governments should save to pay the interest on their public debts.

Mainstream monetarist economics dismissed the supply and demand theory thus adopting the convenient methodology of “models”, a methodology that allows mainstream economists to create convenient models focused on results previously established, based on appropriate assumptions and using suitable mathematical methods. Accordingly, in the case of public debt the mainstream monetarist approach is essentially accounting and hence the result systematically obtained is that there is a primary surplus that provides a condition for the public debt stability. The mainstream proposal is not to ask if in the real world the public debt actually or potentially may converge to an equilibrium level, but to calculate the primary surplus that supposedly assures debt stability thus assuring the monetary policy existence.

The primary surplus condition must be fulfilled by the fiscal policy; it is not a duty of the creators of the public debt. This mandatory primary surplus shall be delivered the next year or in the long run. “The standard debt sustainability assessment frameworks used by international organisations like the European Commission is based on an analysis of debt and debt service dynamics derived from projections of a number of indicators over a medium to long-term horizon” (Veld & o, 2012, 3). Therefore, the stability of the public debt is assured after some time after monetarists provided the “right” projections.

Given the goal of the model, that is, the stable ratio debt/GDP that may be attained through the right economic fiscal policy, three basic ad hoc assumptions are necessary to the monetarist models: 1) at any point in time the previous public debt is “given”, despite the fact that the public debt is a dynamic economic phenomenon, 2) public debt is not affected by the monetary policy; it is a fiscal policy concern and, crucially, 3) GDP and tax revenue are independent from previous public debts and primary surpluses. “Their method was to dismiss the problem from the *corpus* of Economics not by solving it but by not mentioning it”, (Keynes, p. 364). Illustrating, Veld & others propose the following model (Veld & o, 2012, 3):

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = \frac{PD_t}{Y_t} + \left(\frac{B_{t-1}}{Y_{t-1}} * (r_t - g_t) \right) + \frac{SF_t}{Y_t}$$

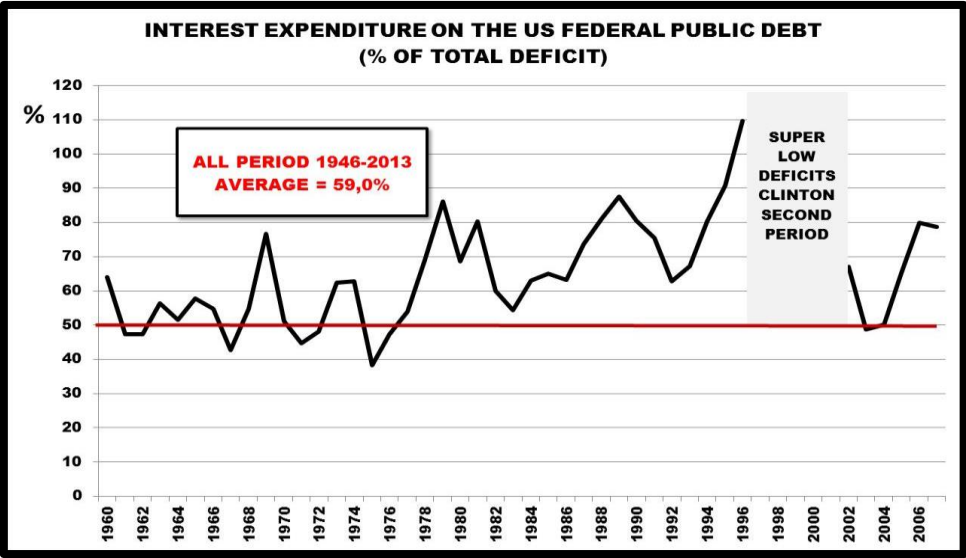
where B/Y is the public debt ratio to GDP, PD is the primary deficit, r is the interest rate, g is GDP growth rate and SF/Y is stock-flow adjustment (a kind of fix-all device?). Debt is a stock, hence this monetarist model is a difference equation, but in the mainstream literature it was traditionally not developed as such. Notwithstanding, monetarists realise that this equation bears a time trend for the term in parenthesis is said to provide a “snow ball” effect on the debt ratio. Mainstream monetarists’ conclusion is that the condition for the debt ratio not to follow an explosive trend is that $g > r$ for the “snow ball” effect becomes negative.

However, monetary policy failure in recovering economies after the 2007 crisis obliged mainstream economists to admit that the monetary policy alone is incapable of restoring the economic activity and some of them manifested concern and doubts about their methods. They

changed their ideas about the multiplier and realized that primary surpluses may cause more deficit and more unemployment (Krugman, 2010; Nautet and Meensel, 2011; Cherif and Hasanov, 2012; Gechert, 2013; Semmler, 2013). One remarkable work was done by Escolano (2010) for it is based and developed using difference equations to prepare the International Monetary Fund’s Practical Guide to Public Debt Dynamics. However, it is just one more impressive mathematical work that looks for the same as ever, the primary surplus that stabilize the public debt ratio. In Escolano words: “Given an initial debt ratio (d_0), and a target debt ratio (d^*_N) to be achieved in N periods, the constant primary balance (p^*) that reaches the target debt ratio if maintained constant during periods $t = 1, \dots, N$ is...” (Escolano, 2010, 5).

Especially worth is the statement by the International Monetary Fund: “This chapter examines the effects of fiscal consolidation—tax hikes and government spending cuts—on economic activity. Based on a historical analysis of fiscal consolidation in advanced economies, and on simulations of the IMF’s Global Integrated Monetary and Fiscal Model (GIMF), it finds that fiscal consolidation typically reduces output and raises unemployment *in the short term*” (IMF, 2010, emphasis added). IMF and others recognise the problem but try to find mitigating situations, for instance talking short and long run, weak economic environment or normal times (Cherif and Hasanov, 2012), low and high debt ratios (Cecchetti, Mohanty and Zampolli, 2011), and anything else that can be used to produce the desired end.

Even mainstream economists can no longer deny that primary surplus decreases GDP thus reducing tax revenue and increasing the public debt. Despite evidences, countries ruled by central banks’ monetary policy keep running primary surpluses and producing negative total results. Accordingly, these countries’ public debts may follow explosive trends (Cherif and Hasanov, 2012; Boussard, 2012). Why primary surplus does not stabilize the public debt ratio? A hypothesis is that the interest expenditure is the most important individual item in the public outlays and deficit and may grow more than the tax revenue, thus creating a rising public deficit. For instance, the figure nearby shows that in United States the interest expenditure accounted for more than half of the federal government deficits in the period 1960-2007, second period of President Clinton aside. During this period 1960-2007 the US federal tax revenue increased yearly at the rate of 7.76% while the interest expenditure rose at 8.36% and the federal public deficit expanded at 6.52%. Therefore, if the intention is to the answer to the question above it seems that the first thing to do is to discard monetarist assumptions and method and search into the possible positive feedback hitting the public debt through the interest expenditure.



The hypothesis in what follows is that, *ceteris paribus* other economic policy instruments, the monetary policy causes GDP and, consequently, tax revenue losses. The test of this hypothesis consists of the estimate of the US GDP and the US federal tax revenue as functions of the main economic policies: fiscal, monetary, income distribution and foreign sector relations. Complementing, it is also presented the estimate of the tax revenue as a function of the GDP. The method follows the non-mainstream approach to the supply and demand theory described in Lima (2015) and tested against reality by an experiment, the estimate of the American aggregate supply curve of national products and services for the period 1960-2007.

3. The US GDP estimate

The estimate of the US GDP obtained in Lima (2015) is:

$$\mathbf{GDPe} = - 277.807 + 6.36696*\mathbf{FE} + 15.5759*\mathbf{ER} - 0.40728*\mathbf{DP} + 3.92613*\mathbf{INT}$$

In this equation **GDPe** is the theoretical equilibrium series² of the US GDP in US\$ billions, **FE** is the US Federal Government fiscal expenses (fiscal policy) in US\$ billions, **ER** is the exchange rate (foreign sector relations policy) in US dollars needed to buy a foreign currencies basket, **DP** is the federal public debt hold by the public and the Federal Reserve in US\$ billions, and **INT** is the interest expenditure on the gross US federal public debt in US\$ billions. Statistical tests indicated that the federal minimum wage (income distribution policy) was never significant (full description of all variables in the Appendix).

Both **DP** and **INT** represent the monetary policy in this context; as expected, **DP** has negative sign and **INT** the positive sign for it is rent transferred to some people who spend the least and save the most. The credit supply thus increases and the aggregate demand expands. But it is not the total interest rent that goes to final products demand. Part of the rent is used to by real estate and financial assets in transactions that only change property but do not create real wealth. Moreover, credit is not free money like government fiscal expenditures produce but interest bearing money; borrowers will somehow take money somewhere to pay the due interest, some demand is cut. Furthermore, the flow of interest rent to Treasury bonds' holders is perennial and is almost always increasing, thus creating an ever growing stock of financial capital and credit supply. Meanwhile borrowers' income available to pay instalments expands slower than the credit supply. Therefore, probably the aggregate demand expansion caused by interest expenditure is restricted.

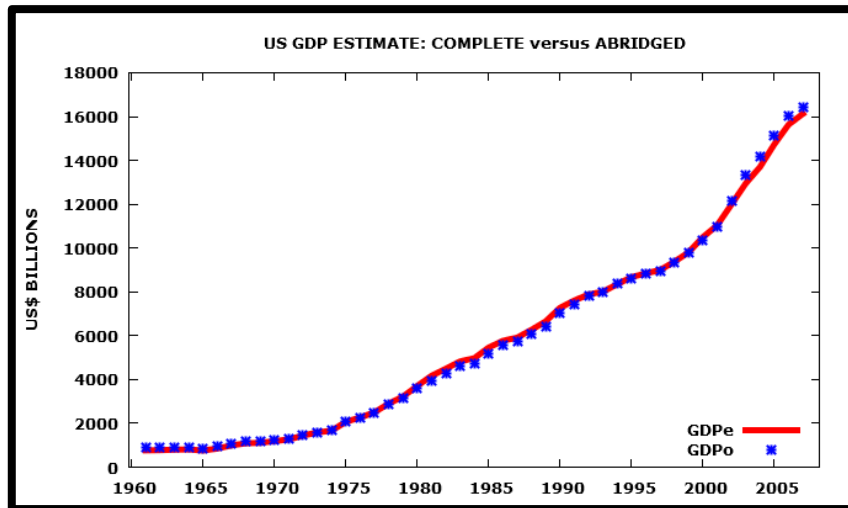
The monetary policy effect looks undefined for the public hold debt **DP** reduces the GDP while the interest expenditure **INT** does the opposite and **INT** is partially a function of **DP**. But it is possible to estimate the combined effect of **DP** and **INT** by omitting **DP** in the GDP equation. This omission imparts a bias to **INT** such that **INT** carries the effect of **DP** and is associated here with the monetary policy applied to the public debt or simply monetary policy. Alternatively, this bias may be introduced without distorting the other coefficients by replacing the **DP** variable in the **GDPe** equation above by its function of **INT** (1960-2007):

$$\mathbf{DP} = -181.112 + 11.012*\mathbf{INT}$$

It is therefore obtained **GDPo**, an abridged version of the US GDP that produces the same estimate of the GDP as shown in the figure nearby:

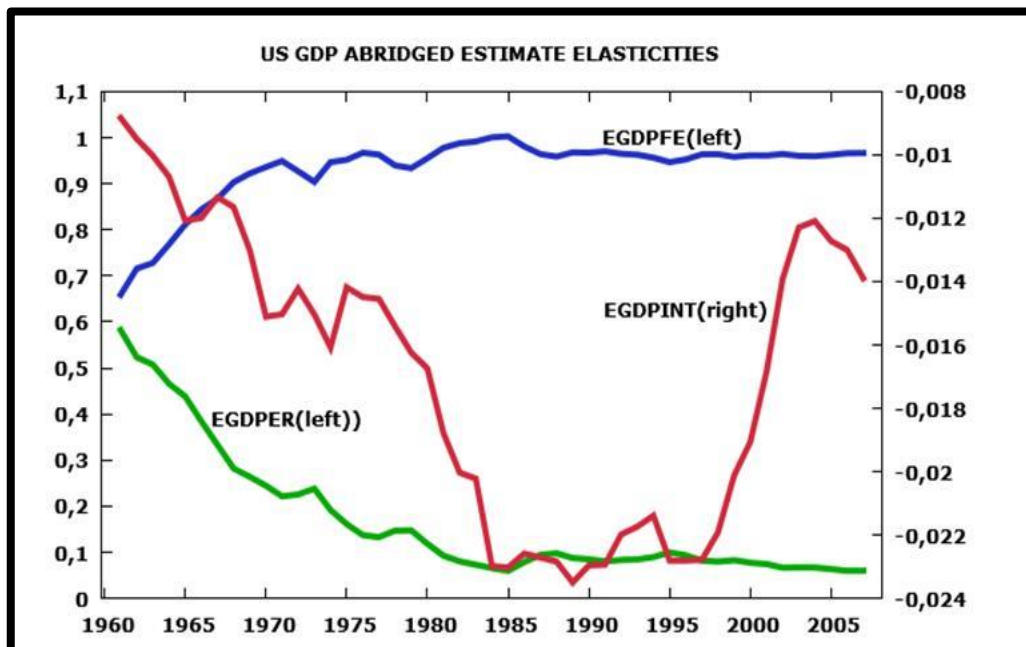
$$\mathbf{GDPo} = - 204.044 + 6.36696*\mathbf{FE} + 15.5759*\mathbf{ER} - 0.5588 *\mathbf{INT} \quad (1)$$

² The theoretical series of an endogenous variable is the series of values that it *would* reach *if* at each point in time all explanatory variables stop varying time enough for their full effects to come about, a condition that is never fulfilled in the real world. Actually, there is no equilibrium between supply and demand for in the real world production takes time (more in Lima, 2015).



This means that if the public debt is allowed to vary INT becomes a measure of the monetary policy and, ceteris paribus the other exogenous explanatory variables, INT bears a negative sign. The combined effect of public debt and its interest rent produces a negative coefficient (-0.5588) and thus the conclusion is that the monetary policy, ceteris paribus the fiscal policy FE and the exchange rate ER, imposed a negative effect on the GDP and on what depends on the GDP.

The past history of the US economic policy mix may be associated with the time performance of the importance of each **GDPo** explanatory variable calculated by its elasticities as shown in the figure nearby. This survey seems to be dependable information for the design of the future policy mix.



This figure shows that the most important variable in the US GDP formation during the period 1960-2007 was the fiscal expenditure for it expanded the aggregate demand without touching the aggregate supply in such a way that its elasticity EGDPF was always positive and the greatest one. On the foreign side, devaluation of the US dollar also created more GDP for the shift to the right it caused to aggregate demand more than compensated the contraction

that the exchange rate imparted to aggregate supply. However, the elasticity EGDPER almost disappeared suggesting that despite the US dollar devaluation the contribution of American exports to the US GDP reduced significantly. The importance of the monetary policy was relatively low and unstable but more public debt continuously reduced GDP for is elasticity EGDPIINT was always negative. Therefore, it may be expected that monetary policy also reduced tax revenue in that period.

4. The US tax receipts estimate

The next step is the estimate of the theoretical equilibrium series of the US federal tax revenue whose reduced equation proposed is:

$$T_t = a + D(L)FE_t + D(L)MW_t + D(L)DP_t + D(L)INT_t$$

where $D(L)$ is a lag operator, T is the federal government current total receipts in US\$ billions, and the exogenous variables explanatory set are the federal government fiscal expenditure FE in US\$ billions, the income distribution policy measured by the US federal minimum wage MW in US\$ dollars per hour, the Federal Government debt hold by the public and the Federal Reserve System DP in US\$ billions, and the Federal Government interest expenditure INT in US\$ billions. Statistical tests revealed that the exchange rate (foreign relations policy) did not influence tax revenue significantly. The estimate obtained for the period 1960-2007 is presented in the Gretl's report nearby.

US FEDERAL TAX REVENUE, 1960-2007					
Dependent variable: T					
	Coefficient	Std. error	t-statistics	p-value	
const	-44.8262	15.5885	-2.8756	0.00637	***
FE_1	0.672873	0.128773	5.2253	<0.00001	***
FE_3	0.513895	0.17324	2.9664	0.00501	***
MW	39.8287	10.4706	3.8039	0.00047	***
DP	-0.957127	0.0395121	-24.2236	<0.00001	***
DP_1	0.899345	0.0439773	20.4502	<0.00001	***
INT_3	0.458746	0.140064	3.2753	0.00215	***

Média var. dependente	865.8853	Dep. var. std. error	726.7012
Soma resíd. quadrados	24001.23	Regression std. error	24.19496
R-quadrado	0.999033	Adjusted R-square	0.998891
F(6, 41)	7059.738	F p-value	3.84e-60
Durbin-Watson	1.739868	DW 1% (6,45):	1.065-1.643

Collecting coefficients for fiscal expenditures FE and public debt DP it is created the theoretical equilibrium reduced equation of the United States Federal Government tax receipts T_e for the period 1960-2007:

$$T_e = -44.8262 + 1.1868*FE + 39.8287*MW - 0.0578*DP + 0.45875*INT \quad (2)$$

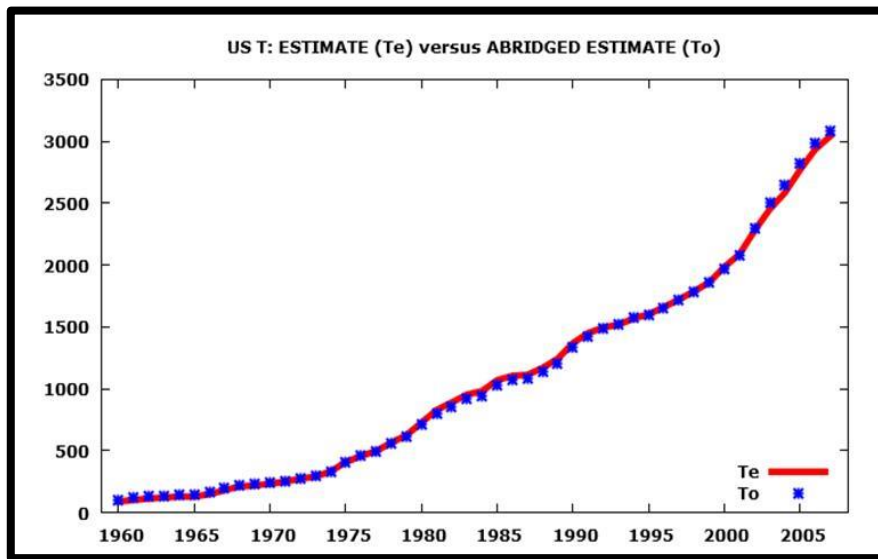
The US tax revenue responded positively to the fiscal policy (FE) and to MW , the income distribution policy indicator. Monetary policy, given by the combination of DP and INT , also has a negative effect on the tax revenue. In fact, replacing DP in the T_e equation by its equation above:

$$DP = -181.112 + 11.012*INT$$

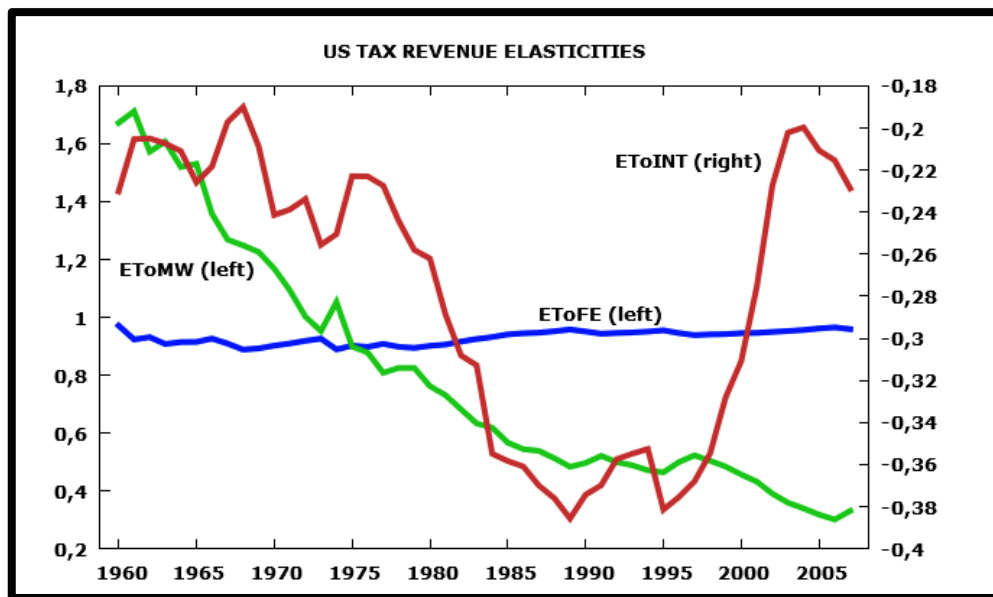
It is obtained the abridged version of the US tax revenue theoretical equilibrium equation:

$$T_o = -34.3679 + 1.1868*FE + 39.8287*MW - 0.1777*INT \quad (3)$$

The figure below shows that both equations (2) and (3) produce the same estimate of the US federal tax revenue series.



The derivative of the tax receipts T in relation to the fiscal expenses FE (≈ 1.19) may be seen as the financial “return coefficient” of the social investment made through fiscal policy. Therefore, ceteris paribus MW , ER , DP and INT government fiscal expenditure cuts cause GDP and thus T to fall and therefore public debt DP and INT to grow. The “return coefficient” of the monetary policy is negative (-0.1777). This result allows for not rejecting the hypothesis that the monetary policy measured by the interest expenditure associated with the public debt may cause a negative effect on the tax revenue. The time performance of the importance of the monetary policy and the other economic policy instruments in the US tax revenue formation, calculated by their elasticities, is shown in the figure nearby.



The most important exogenous variable in the T_o formation was the fiscal policy for its elasticity E_{ToFE} was stable and the highest one since 1975. The elasticity of T_o in relation to the minimum wage E_{ToMW} presents a fading performance, what is evidence that this

instrument of income distribution policy despite its positive effect was neglected. On the negative side, the monetary policy as measured by merging **DP** and **INT** suggests that the positive effect to the tax revenue allowed by the credit supply was insufficient to offset the negative effect provided by the public debt itself. Actually, the monetary policy effect given by EToINT despite unstable is always negative and relevant. The conclusion is thus that this experiment does not allow for the rejection of the hypothesis that monetary policy may cause tax receipts losses.

Actually, the endogenous variable tax revenue is a function of another endogenous variable, the GDP as a measure of the tax base. The estimate of a relation between two or more endogenous variables requires that all exogenous variables of all endogenous variables but one be in the explanatory set of the dependent variable and that the endogenous variables be measured by their theoretical equilibrium series. The exogenous variable omitted will then be the only one that makes all endogenous variables vary thus revealing one relation among them; different exogenous variable omitted produces different slope coefficients (Lima, 2015).

Accordingly, in this experiment the estimate of the tax revenue **T** as a function of **GDP** was made using the theoretical equilibrium series, respectively **Te** (equation 2) and **GDPe** (equation 1) and the exogenous variable omitted is the fiscal expenditure **FE**. While the other variables are kept under the ceteris paribus condition fiscal expenditure **FE** varies thus creating the line of points (**Te**, **GDPe**) in the orthogonal plan **GDPe** x **Te**. This line may be obtained by algebra, but in order to avoid mistakes it was run the regression presented in the Gretl's report below in which statistical results are of course meaningless.

US FEDERAL TAX REVENUE as a function of GDP, 1961-2007					
Dependent variable: Te					
	Coefficient	Std. error	t-statistics	p-value	
const	6.95697	5.121e-012	1.35845E+12	<0.00001	***
GDPe	0.1864	0	2.13411E+14	<0.00001	***
MW	39.8287	2.3523e-012	1.6932E+13	<0.00001	***
ER	-2.90334	1.7762e-013	-1.63457E+13	<0.00001	***
DP	0.0181169	0	4.51458E+12	<0.00001	***
INT	-0.27308	0	-9.29123E+12	<0.00001	***
Dependent var. avg.		1095.627	Dep. var. std. error		852.8918
Square residuals sum.		8.36e-22	Regression std. error		4.52e-12
R-square		1.000000	Adjusted R-square		1.000000

The “structural” US tax revenue equation thus obtained is:

$$\mathbf{Te = - 6.9570 + 0.1864*GDPe + 39.8287*MW - 2.90334*ER + 0.01812*DP - 0.2731*INT}$$

Once again, merging DP and INT creates the abridged US tax revenue equation:

$$\mathbf{To = - 10.239 + 0.1864*GDPe + 39.8287*MW - 2.90334*ER - 0.07356*INT}$$

This estimate adheres to the idea that GDP is a measure of tax base. However, it is not possible to apply the ceteris paribus condition to an endogenous explanatory variable for its variations are associated with variations in at least one of the exogenous variables that simultaneously produce variations in the dependent endogenous variable. This means that this equation is not a safe indicator of the tax burden and the relevance of the exogenous variables in the tax revenue formation. Actually, the analysis of the causes of endogenous economic events must be based on the theoretical equilibrium reduced equations of endogenous

variables associated with such events. Nevertheless, empirical data suggests not discarding the idea that the interest expenditure combined with the public debt causes a negative effect on the tax revenue. The main conclusion is that this experiment does not allow for rejecting the hypothesis that monetary policy causes GDP and, consequently, tax revenue losses.

Therefore, it seems that there is a positive feedback to the public debt: more public debt leads to more interest expenditure and to lower tax revenue that leads to larger deficit and to more public debt and so on. The next point is the demonstration that this feedback imparts an explosive trend to the public debt.

5. A theorem for the public debt time trend

This section presents a dynamic non-mainstream approach to the public debt demonstrating that it may follow an explosive trend, possibly as a natural economic fact. Suppose initially that there is neither money emission to pay the interest or the principal, nor public debt created by pure monetary policy³. So, once initiated the public debt levels result only from the accumulation of previous deficits. Therefore, the time performance of the public debt may be described by the following accounting-looking expression:

$$D = D_{-1} + FE + INT - T \quad (4)$$

where D is the public debt at the end of the current period, D_{-1} is the public debt at the end of the previous period, FE is the government's fiscal expenditure, INT is the government's expenditure on interest and T is the tax revenue. All values are nominal and not ratios to GDP. Defining the interest expenditure INT as the product of the exogenous variable interest rate r by the public debt at the end of the previous year D_{-1} it comes:

$$INT = r D_{-1} \quad (5)$$

Next, substituting INT in the accounting-looking expression (4) and rearranging one obtains:

$$D = (1 + r) D_{-1} + FE - T \quad (6)$$

The expression that describes the public debt is no longer an accounting-looking one; it is a linear difference equation since debt is a matter of economics. Accordingly, the tax receipts T is not given nor assumed nor predicted, but an endogenous variable, a straight function of the tax base measured by the GDP and the tax rate imposed by government. Given the experiment above, and especially the equation (3), it is expected that the combined effect of the public debt (D and $r \cdot D_{-1}$) over GDP and hence on the tax earnings T be negative. It may thus be assumed that there are evidences supporting the assumption that the tax receipts may be generically described by a function like:

$$T = \beta_0 + \beta_1 FE + \beta_2 INT + \beta_3 OV \quad (7)$$

where β_1 is the return coefficient of the fiscal policy, or the measure of the return to the government of the social investment it made through fiscal expenditures FE , and it is expected to be positive. β_2 is the return coefficient of the monetary policy, or the measure of the return to the government of the public debt it made, and it is expected to be negative (equation 3). OV refers to other exogenous variables except D ; for instance, in this US case OV contains only the federal minimum wage MW .

³ A monetary policy is here said to be "pure" in the cases it refers to money supply or inflation control, quantitative easing and others; public deficit financed with Treasury bonds emission is not pure monetary policy but a condition for pure monetary policy to exist.

Replacing T (7) and INT (5) in the expression of the public debt (6) and rearranging it comes:

$$D - \{1 + r(1 - \beta_2)\} D_{-1} = -\{\beta_0 + (\beta_1 - 1)FE + \beta_3 OV\} \quad (8)$$

This is a difference equation that describes the time behaviour and the possible theoretical equilibrium levels of the public debt. The theoretical equilibrium solution for this function is given by:

$$D^* = 1/\{r(1 - \beta_2)\} \{\beta_0 + (\beta_1 - 1)FE + \beta_3 OV\} \quad (9)$$

This is a complex solution, stressing that the hypothetical equilibrium level of the public debt D^* depends on the monetary policy's exogenous variable interest rate r , on the coefficient of the monetary policy β_2 , on the social investment return β_1 , on the exogenous variable fiscal expenses FE , and on other exogenous variables OV . Under normal conditions the hypothetical equilibrium level of the public debt D^* is expected to be positive and not constant; the theoretical equilibrium status of the public debt is always moving. Complementing, the trend solution is expressed by:

$$D_{\text{trend}} = A [1 + r(1 - \beta_2)]^t \quad (10)$$

where A is a constant to be determined. The twofold condition for the public debt to be convergent to a theoretical equilibrium status is:

$$0 < \{1 + r(1 - \beta_2)\} < 1$$

Taking one of the inequalities:

$$\{1 + r(1 - \beta_2)\} > 0$$

and transposing terms one obtains the first condition for convergence:

$$r > -1/(1 - \beta_2) \quad \text{or} \quad \beta_2 < (1/r)(1 + r)$$

This inequality may hold or not for alternative pairs of r and β_2 . Particularly, if as expected from the experiment above β_2 is negative while r is positive, then any pair of simultaneous normal values of r and β_2 satisfies the first condition. Complementing, the second condition for convergence towards a stable public debt level is:

$$\{1 + r(1 - \beta_2)\} < 1, \quad \text{implying that} \quad r(1 - \beta_2) < 0.$$

This inequality may hold only if the interest rate r and $(1 - \beta_2)$ have simultaneously opposite signs. This means that the public debt may converge towards a stable level only in one of two possibilities. In the first one the interest rate r is positive, and thus β_2 must be positive and greater than one, meaning that the tax receipts must be positively associated with the interest expenditure on public debt. The second possibility is that the public debt causes a so huge negative effect on the economic activity that the credit expansion allowed by the interest on the public debt cannot compensate it and hence the net effect of the monetary policy is a reduction of the tax receipts; that is, β_2 is negative. In this case $(1 - \beta_2)$ is positive and thus equilibrium requires that the interest rate r upon the public debt be negative.

The conclusion is firstly that if as expected from the experiment above β_2 is negative while r is positive then no pair of simultaneous values of r and β_2 can satisfy the second condition. This means that the product $\{r(1 - \beta_2)\}$ is always positive and that the partially autonomous trend component (10) leads public debt to infinity. Depending on the capacity of the public debt in creating tax revenue any public debt may carry an explosive trend. Secondly, data demonstrates that the US public debt was following an explosive trend until 2007. Thirdly, the debt trend (10) depends exclusively upon β_2 , the effect of interests over the

tax receipts, and r , the interest rate fixed by the central bank. Hence, primary surplus does not touch the public debt trend; if β_2 is negative primary surplus cannot provide stability to the public debt and hence to the debt ratio. All the same, privatisation does not touch public debt's trend but reduces the fiscal expenses and raises the implicit tax rate thus reducing GDP and tax revenue. Fourthly, dividing public debt by GDP changes nothing for GDP depends ultimately on natural resources and cannot grow to infinity.

If the US case can be generalized, once initiated public debt keeps always growing to infinity; it is out of the control of the mainstream monetarist economic policy. Falsifying this conclusion requires the proof that in dependable cases β_2 is positive, i. e. that an increase in the interest rent on the public debt normally expands tax revenue.

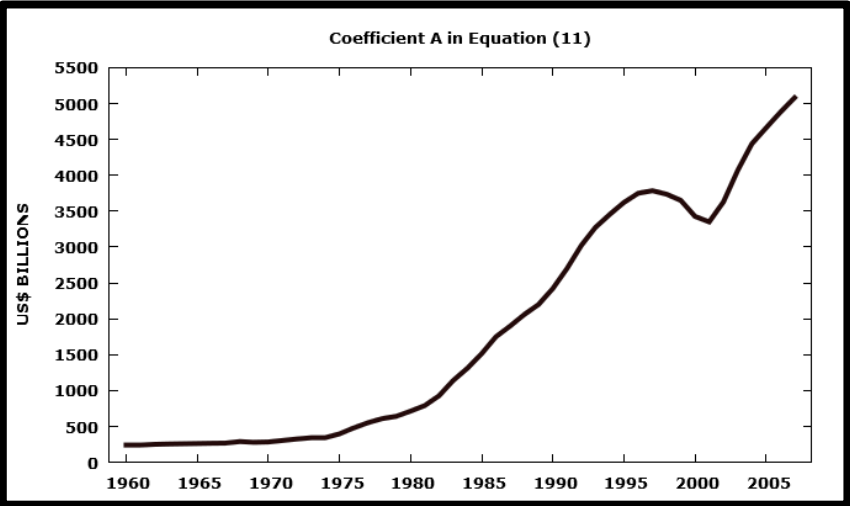
The general solution of the public debt time performance is given by the summation of theoretical equilibrium and trend solutions:

$$D_t = A [1 + r (1 - \beta_2)]^t + D^*$$

The value of the coefficient A is found associating the value of D_t to the point zero in the time series, hence obtaining: $D_{t=0} = A + D_0^*$ and thus $A = (D_{t=0} - D_0^*)$. Therefore, the complete equation that describes the time performance of the public debt is:

$$D_t = A (D_{t=0} - D_0^*) [1 + r (1 - \beta_2)]^t + D^* \tag{11}$$

The sign of the coefficient A gives the direction that D_t will follow. For instance, it may be expected that in case of debts normally $A > 0$ for at each point in time the theoretical equilibrium of any debt should be lower than the actual one. In this US case the figure nearby shows that the coefficient A was positive as expected but growing, revealing that the distance between the present and equilibrium increased, that is, in dollars the theoretical equilibrium was further and further away.



Both theoretical equilibrium D^* and interest rate r are not constants; they are time series that depend on economic policy decisions and performance and on other exogenous variables that are changing continuously. Equation (11) reveals the nature of the phenomenon but the trend slope and speed are also always changing. Hence, there is an equation (11) for each point in time that could trace out the future time behaviour of the public debt if economic policy and exogenous variables stop varying; actually, each point is not a single debt value but an equation. What matters is that always that the interest rate is positive and β_2 is negative the coefficient $[1 + r (1 - \beta_2)]$ will be greater than one and consequently the public debt D_t will follow an explosive trend towards infinity.

About β_2 , in a private investment project interest expenses push debt upwards and return on the financial capital invested pushes debt down. The return must be positive and greater than the expenditure on interest; a coefficient like β_2 must be positive. If so, then the private debt will follow a trend to zero; otherwise the debt will follow an explosive trend to infinity, the project will bankrupt and stockholders will lose their income and, partially or entirely, their wealth. The same principle applies to the public debt; money borrowed by government must generate tax revenue sufficient to pay the due interest. In the case β_2 is negative the primary surplus is systematically lower than the interest to be paid. Consequently, people may lose their income and, partially or entirely, their wealth. Of course, ethics oblige economists to disapprove economically unfeasible private investment projects, but the same principle has not been applied to the public investment that creates jobs and social wealth.

About interest rate, negative values mean money emission and money may be issued by private central banks or by governments directly. Central banks create money out of thin air when paying the interest rent on Treasury bonds they redeem. So, money that pays interest on Treasury bonds is unbacked money. This unbacked money is potentially inflationary especially of assets like for instance houses, directly or via credit; this unbacked money emission may be at the origin of financial crises. On the opposite direction, money created out of thin air by governments is backed money for it is issued to buy goods and services produced by people thus expanding GDP and employment. Of course in this case prices also rise but income rose first, people escalated the Maslow's Pyramid and that is fine.

About the theorem, are unfeasible the assumptions that there is no money emission neither to pay interests nor to amortize the debt, and that there is no pure monetary policy. This experiment refers to the case of public debt resulting only from the accumulation of previous governmental deficits but actually monetary policy may increase the public debt when asking Treasury to issue bonds to operate the open market and reduce it by printing money to redeem Treasury bonds presented by the Treasury (seigniorage). These are casual autonomous events that cannot be anticipated neither in time nor in extension. Considering however that central banks are profit-seeking private business, the probability of bonds issuing is very much greater than money printing to reduce or avoid the public debt. Consequently, it seems that the probability of public debt explosions everywhere is not to be disregarded.

6. Conclusions

The mainstream approach to the public debt is the same one dedicated to the private companies and consumers; mainstream economists do not study public debt from a macroeconomic standpoint. Mainstream economists are convinced that there is always a feasible primary surplus that government can produce to stabilize the public debt ratio to GDP.

This study shows that the public debt may have an equilibrium level but suggests that the convergence to it looks improbable for it depends on how much the money borrowed by government can expand aggregate demand and thus increase tax receipts. The estimate of the US federal tax revenue points to a positive effect of the interest rent on the US tax receipts that is insufficient to compensate the simultaneous negative effect of the public debt itself. Thus, considering that these variables are mutually dependent, the conclusion is that the combined result is negative. Therefore, at least in the period 1960-2007 the US public hold debt was following an explosive trend. The immediate consequence is that the unbacked money issued by central banks without auditing to pay interest on Treasury bonds followed the same trend to infinity.

The financial, economic and social crisis that came next was predictable for the same explosive tendency applies, besides unbacked money emission, to everything else touched by

public debt: interest rent to the few, private excessive savings, money supply excessive expansion, financial assets inflation, real estate prices inflation, financial crises, frauds, corruption, bail outs, monetary policy crises, income and wealth concentrations, food prices inflation, unemployment, and much more. The main conclusion is that preventing public debt explosive trend and avoiding its consequences is not an exclusive mainstream monetary concern. It is missing deeper non-mainstream studies of the monetary policy questioning its scientific validity for it is possible that it is one important root of the great economic and social problems real world people are facing.

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Appendix

Variables: definitions and sources.

Data are current values.

T - Federal government; current total receipts, US\$ billions, White House, Office of Management and Budget, Historical Tables, 2014, Tables 1.4.

<http://www.whitehouse.gov/omb/budget/Historicals>.

FE - Federal Government, Fiscal Expenditure, estimated by subtracting Net interest expenditure from Total outlays, US\$ billions, White House, Office of Management and Budget, Historical Tables, 2014, Tables 1.4 and 3.1.

<http://www.whitehouse.gov/omb/budget/Historicals>.

ER - Exchange rate, average exchange rate in US dollars needed to buy a foreign currencies basket weighted by the corresponding US exports. Original data: OECD. Sample: Austria, Belgium, Canada, China, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, Switzerland and United Kingdom.

http://stats.oecd.org/Index.aspx?DataSetCode=SNA_TABLE4.

MW - Federal minimum wage, US\$ per hour, U.S. Department of Labor.

<http://www.dol.gov/whd/minwage/chart.htm>.

DP - Federal Government debt hold by the Federal Reserve System and the public, US\$ billions, White House, Office of Management and Budget, Historical Tables, Table 7.1.

<http://www.whitehouse.gov/omb/budget/Historicals>.

INT – Federal Government interest expenditure, US\$ billion, Federal Reserve System, Data Download Program.

<http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1>.